NOTATION

T, substrate temperature; T_0, initial substrate temperature; r and z, radial and axial coordinates; t, time; Q, heat-radiation source power; λ, thermal conductivity; a, thermal diffusivity; r_0, radius of the heat radiation beam; J_0 and J_1, Bessel functions of the zero and first order; Φ, error integral; ΔT, excess temperature; T_{st}, stationary temperature; β, local speed of response; $\tau_{1/2}$, time required for reaching half of the maximum excess temperature; C_p, specific heat; ρ, density.

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NUMERICAL METHOD FOR ANALYZING A STOCHASTIC STATIONARY HEAT-CONDUCTION EQUATION WITH RANDOM COEFFICIENTS

A. G. Madera

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A numerical method is suggested for defining mathematical expectation fields and the variance of a stochastic temperature field which is described in the stationary case by a stochastic heat conduction equation and boundary conditions with random coefficients. Random coefficients of the stochastic mathematical model may obey arbitrary truncated distribution laws. An example of using the developed method is presented.

Introduction. Real temperature distributions in real objects are stochastic. This fact is caused by the randomness of the parameters and characteristics determining a temperature field. Such parameters and characteristics as powers of sources and sinks of heat, thermal conductivity coefficients of a body, coefficients of heat transfer from a body surface into a medium, environment temperature, gaps between contacting bodies, etc., may be random and have a significant statistical scatter. The stochasticity of these parameters and characteristics is a consequence of the random technological scatter and random fluctuations of the parameters characterizing heat transfer between the object and the medium.

In engineering practice the temperature mathematical-expectation and temperature variance fields are the most important probability characteristics of the stochastic temperature distribution in objects. Having available these probability characteristics, one can determine the fields of confidence intervals in the object. The real values of temperatures (which may occur in practice) will be arranged inside these intervals.

At present, there exist the following numerical methods for analyzing stochastic temperature fields in a body: perturbation theory methods [1]; the finite-element method for a differential equation with the coefficient of an unknown and the free term both being white Gaussian noise [2]; and the method of the stochastic Green's function [3, 4]. However, the perturbation theory methods are applicable only in the case when random fluctuations of the parameter are much smaller than

Institute of Cybernetics Problems of the Academy of Sciences of USSR, Moscow.

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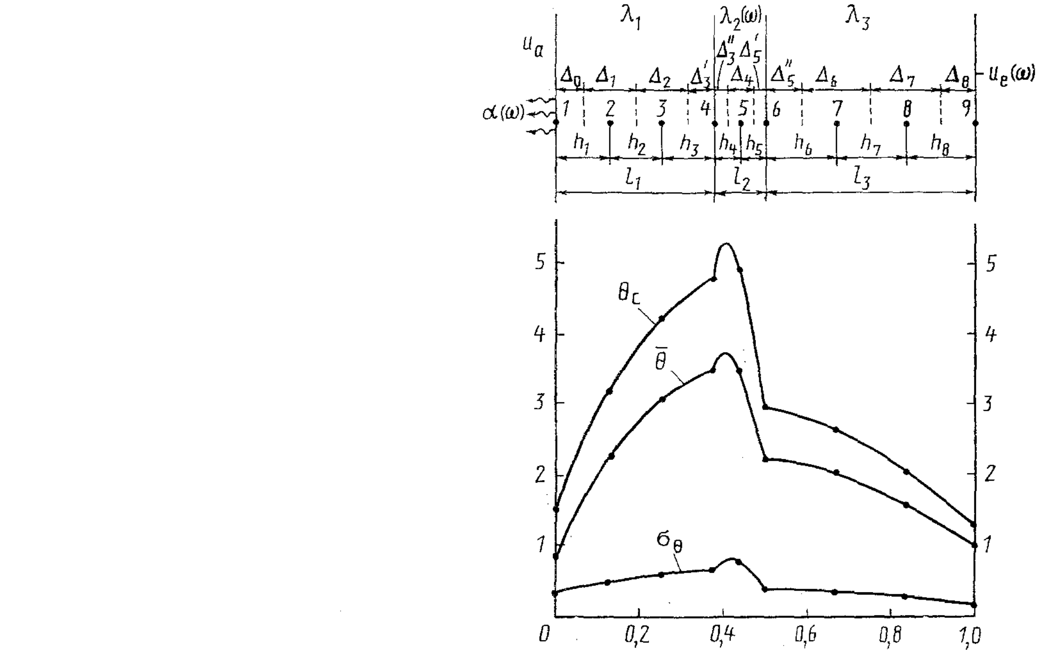

Fig. 1. Dimensionless distributions of the mathematical expectation $\bar{\Theta}$, the r.m.s. deviation σ_Θ and the upper 95% confidence boundary Θ_c for the stochastic temperature field in a three-layer plate.

its mathematical expectation value. In the finite-element method [2], one considers the equation with the zero first-order boundary condition and the deterministic coefficient of the second derivative. Besides, this method is developed for the randomnesses of the white Gaussian noise type, which fit the real randomnesses not always well. The method of the stochastic Green's function is used only when Green's function may be constructed analytically, i.e., for a very narrow class of problems. There are no effective numerical methods that we needed for the analysis of stochastic three-dimensional temperature fields in regions of complex form which are described by the stochastic heat conduction equation and arbitrary boundary conditions with random coefficients, obeying real distribution laws.

In the present work we propose a numerical method for defining mathematical expectation and variance distributions of the stochastic temperature field in a body of any dimensionality which is described by the stochastic stationary heat conduction equation in terms of partial derivatives. All the coefficients that enter into the equation and into the boundary conditions are random and obey arbitrary, truncated distribution laws. The probability characteristics of the stochastic temperature field are derived in an analytical matrix form. The method is based on an application of technique [5] developed by the authors to a system of stochastic matrix equations that were obtained after approximating an operator in partial derivatives and boundary conditions by their difference analog. The error of the method is evaluated by the difference approximation error of the equations of the mathematical model and the discretization region.

Stochastic Mathematical Model. The mathematical model describing the stochastic stationary temperature distribution u(x, ω) in the three-dimensional region D from R^3 with the boundary ∂D takes on the form

$$\nabla \cdot (\lambda(x, \omega) \nabla u(x, \omega)) + f(x, \omega) = 0, \; (x, \omega) \in D \times \Omega, \tag{1}$$

with boundary conditions on ∂D of one of the three forms:

$$u(x, \omega) = f_1(x, \omega), \; (x, \omega) \in \partial D \times \Omega, \tag{2}$$

$$\lambda(x, \omega) \frac{\partial u(x, \omega)}{\partial n} = f_2(x, \omega), \; (x, \omega) \in \partial D \times \Omega, \tag{3}$$

$$\lambda(x, \omega) \frac{\partial u(x, \omega)}{\partial n} + \alpha(x, \omega)(u(x, \omega) - f_3(x, \omega)) = f_2(x, \omega), \; (x, \omega) \in \partial D \times \Omega, \tag{4}$$

where $x = (x_1, x_2, x_3) \in D$; $\lambda(x, \omega) > 0$, $f(x, \omega)$, $f_i(x, \omega)$, $i = 1, 2, 3$, and $\alpha(x, \omega) > 0$ are assigned functions of $x \in \bar{D} = D + \partial D$ which for each $x \in \bar{D}$ are random independent quantities.

In real bodies and in their systems the enumerated random functions at any $x \in \bar{D}$ and $\omega \in \Omega$ vary within limited ranges. Therefore, their probability densities are truncated, i.e., continuous within the intervals of variation of random functions and equal to zero outside these intervals.

The random functions that enter into the mathematical model simulate: $f(x, \omega)$ — the volumetric distributed heat source with a random volumetric power density; $f_1(x, \omega)$ - the random temperature prescribed at the region boundary ∂D; $f_2(x, \omega)$ — the random surface power density at the region boundary; $f_3(x, \omega)$ — the random value of the surrounding medium's temperature; and $\alpha(x, \omega)$ — the random coefficient of heat transfer from the body surface into the medium.

Difference Approximation. For definiteness and clarity of formulation, we consider the difference approximation of the stochastic heat conduction equation (1) with boundary condition (2) in the two-dimensional rectangular region $\bar{D} = \{0 \le x \le l_1$ and $0 \le y \le l_2\}$. Introduce into $\bar{D}$ a rectangular non-uniform grid with steps equal to h_i^x, i = 1, 2 ..., N + 1 along the x-axis and h_j^y, j = 1, 2 ..., M + 1 along the y-axis. Nodes with numbers 0, N + 1, and M + 1 lie at the region boundary ∂D.

The difference scheme for Eqs. (1) and (2), which is derived by the integro-interpolation method [6], is valid for each $\omega \in \Omega$ and has the form

$$\delta_{ij}u_{i-1,j} - (\delta_{ij} + \delta_{i+1,j} + \varepsilon_{ij} + \varepsilon_{i,j+1})u_{ij} + \delta_{i+1,j}u_{i+1,j} + \varepsilon_{ij}u_{i,j-1} + \varepsilon_{i,j+1}u_{i,j+1} + f_{ij} = 0, \tag{5}$$

$$u_{0j}(\omega) = f_1(x_0, y_j, \omega), \; u_{N+1,j}(\omega) = f_1(x_{N+1}, y_j, \omega), \; j = 0, 1, \ldots, M+1, \tag{6}$$

$$u_{i0}(\omega) = f_1(x_i, y_0, \omega), \; u_{i,M+1}(\omega) = f_1(x_i, y_{M+1}, \omega), \; i = 0, 1, \ldots, N+1,$$

where $u_{ij} = u_{ij}(\omega) = u(x_i, y_j, \omega)$ is the stochastic temperature at the node i, j;

$$\delta_{ij} = \delta_{ij}(\omega) = \lambda_{i-\frac{1}{2}}(\omega)\frac{\Delta_j^y}{h_i^x}, \quad \varepsilon_{ij} = \varepsilon_{ij}(\omega) = \lambda_{i,j-\frac{1}{2}}(\omega)\frac{\Delta_i^x}{h_j^y},$$

$$f_{ij} = f_{ij}(\omega) = \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} f(x, y, \omega)\,dxdy. \tag{7}$$

After moving the known boundary values of temperatures and free terms over to the right-hand side, one may write (5) in the form of a system of stochastic linear algebraic equations

$$R(\omega)u(\omega) = -f(\omega) - \varphi(\omega), \tag{8}$$

where $u(\omega) = (u_{11} \ldots u_{N1}, u_{12} \ldots u_{N2}, , u_{1M} \ldots u_{NM})^T$ is the stochastic vector of unknown temperatures at internal grid nodes; $R(\omega)$ is the stochastic three-diagonal symmetric n × n matrix (n = NM) with the block structure:

$$R(\omega) = \begin{bmatrix} \delta_1 & \varepsilon_2 & & & \\ \varepsilon_2 & \delta_2 & \varepsilon_3 & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \varepsilon_M \\ & & & \varepsilon_M & \delta_M \end{bmatrix},$$

in which matrices $\delta_i = \delta_i(\omega)$, i = 1, 2, ..., M are stochastic tri-diagonal symmetric matrices with the diagonal elements equal to $-(\delta_{ki} + \delta_{k+1,i} + \varepsilon_{ki} + \varepsilon_{k,i+1})$, k = 1, 2, ..., N and with the elements that are symmetric relative to the diagonal equal to δ_{li}, l = 2, 3, ..., N; matrices $\varepsilon_i = \varepsilon(\omega)$, i = 1, 2, ..., M are the stochastic diagonal ones with elements ε_{ki}, k = 1, 2, ... N; and $f(\omega)$ and $\varphi(\omega)$ are stochastic dimensional vectors of length n consisting of heat powers at grid nodes and of the known temperatures at the region boundary, respectively.

We represent the matrix $R(\omega)$ and vectors $f(\omega)$ and $\varphi(\omega)$ as:

$$R(\omega) = -AG(\omega)A^T = -H(\omega), \tag{9}$$

$$f(\omega) = Af'(\omega), \tag{10}$$

$$\varphi(\omega) = -AG(\omega)\varphi'(\omega), \tag{11}$$

where A is a deterministic rectangular n × m matrix (m = 2NM + N + M) with the block structure:

$$A = \begin{bmatrix} T & -E & 0 & E & & & & & \\ & & T & -E & 0 & E & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & & T & -E & 0 & E & \\ & & & & & & T & -E & E \end{bmatrix},$$

T is a deterministic N × (N + 1) matrix of the form

$$T = \begin{bmatrix} 1 & -1 & & & & \\ & 1 & -1 & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & 1 & -1 & 0 \\ & & & & 1 & 1 \end{bmatrix},$$

E is a diagonal N × N unit matrix; 0 is a zero N × (N + 1) matrix; $G(\omega)$ is a stochastic diagonal m × m matrix with a block structure with element-matrices arranged along the diagonal in the following order: δ_1, ε_1, δ_2, ε_2, ... , δ_M, ε_M, and ε_{M+1}; $f'(\omega)$ and $\varphi'(\omega)$ are stochastic m-dimensional vectors.

In view of (9)-(11), the set of (8) will appear as follows

$$H(\omega)u(\omega) = Af'(\omega) - AG(\omega)\varphi'(\omega). \tag{12}$$

The elements $g_{ii}(\omega)$ of the stochastic matrix $G(\omega)$ are assumed to be random quantities with different truncated distribution laws. The elements g_{ii} are statistically independent of each other and of the stochastic vectors $f'(\omega)$ and $\varphi'(\omega)$. The vectors $f'(\omega)$ and $\varphi'(\omega)$ may be statistically dependent. Note that the condition $|g_{ii} - \bar{g}_{ii}|/\bar{g}_{ii} < 1$ (where $\bar{g}_{ii} = M\{g_{ii}(\omega)\}$ is the mathematical expectation of the random quantity $g_{ii}(\omega)$) is always satisfied in practice.

The difference approximation of the mathematical-model equations leads to matrix equation (8) with the symmetric matrix R irrespective of the dimensionality of the equations and of the region's form. A representation of the matrix R in the form of the product of three matrices (9), in the middle of which there is the diagonal matrix G, and of the vectors f and φ in the form of (10) and (11) may always be realized. In this case, the diagonal matrix G contains elements determined in terms of the thermophysical parameters of the region and boundary conditions, while the matrix A consists of elements 1, -1, and 0. The matrix structure A is determined by the problem's dimensionality and by the form of boundary conditions. It is easy to understand the existence of the representation (9) if we interpret the grid covering the region as a graph with vertices at grid nodes and with branches connecting these nodes, containing thermal resistances or conductances. Then, according to the matrix-topological theory of electrical circuits [7], it is always possible to number the nodes and branches of the graph so as to obtain the matrix representation of the system R in the form of (9).

Determination of the Probability Characteristics. We determine the probability characteristics of stochastic temperatures at grid nodes, namely: the mathematical expectation vector $\bar{u} = M\{u(\omega)\}$ and the correlation matrix $K = M\{u(\omega)u^T(\omega)\}$, where $u(\omega)$ is calculated from Eq. (12): $u(\omega) = H^{-1}(\omega)\{Af'(\omega) - AG(\omega)\varphi'(\omega)\}$; $M(\cdot)$ is the mathematical expectation operator. The covariance matrix C is predicted from the expression $C = M\{u^0(\omega)(u^0(\omega))^T\} = K - \bar{u}\bar{u}^T$, and the vector of variances $D = M\{(u^0(\omega))^2\}$, $u^0(\omega) = u^0(\omega) - \bar{u}$, of stochastic temperatures at the grid nodes is equal to the diagonal elements of matrix C.

The stochastic matrix equation (12) with the stochastic matrix $H(\omega) = AG(\omega)A^T$ was investigated in [5]. The reduced expressions in [5] for vector $\bar{u}$ and matrix K of stochastic temperatures at the nodes of the equivalent electrical circuit are obtained under the assumption that $M\{(g_{ii}^0)^k(g_{jj}^0)^l\}$ are negligible for all $i \neq j$, k, $l \geq 2$; the comparison of u and K with the calculations, performed by the Monte-Carlo method, showed their good agreement. If we do not neglect the mutual moments of the quantities (g_{ii}^{0k}), i = 1, 2 ..., m, $k \geq 2$, then the probability characteristics of the stochastic vector take on the form: for the numerical expectation vector

$$\bar{u} = B(E - \bar{W}F)(\bar{f}' - \bar{G}\bar{\varphi}') - B\bar{W}\bar{\varphi}'; \tag{13}$$

and for the correlation matrix

$$K = B\{\bar{Z}_1 - \bar{Z}_1F\bar{W} - \bar{W}F\bar{Z}_1 - \bar{W}\bar{Z}_2 - \bar{Z}_2^T\bar{W} + M\{W\bar{Q}W\}\}B^T, \tag{14}$$

where $\bar{f}' = M\{f'(\omega)\}$, $\bar{\varphi}' = M\{\varphi'(\omega)\}$, $\bar{G} = M\{G(\omega)\}$; $\bar{Z}_1 = M\{\varphi(\omega)\varphi^T(\omega)\}$, $\bar{Z}_2 = M\{\varphi'(\omega)\varphi^T(\omega)\}$, $\bar{Z}_3 = M\{\varphi'(\omega)\varphi'(\omega))^T$, and $\varphi(\omega) = f'(\omega) - \bar{G}\varphi'(\omega)$ are deterministic matrices; $\bar{W}$ is a deterministic matrix which is equal to the mathematical expectation of the matrix stochastic series, i.e., $\bar{W} = M\{G^0(\omega) - G^0(\omega)FG^0(\omega) + G^0(\omega)FG^0(\omega)FG^0(\omega) - \ldots\}$; $\bar{Q} = F\bar{Z}_1F + \bar{Z}_2F + F\bar{Z}_2^T + \bar{Z}_3$ is a deterministic symmetric $m \times m$ matrix; and $B = (A\bar{G}A^T)^{-1}A$, $F = A^TB$ are deterministic $n \times m$ and $m \times m$ matrices, respectively.

The matrix series $\bar{W}$ converges almost surely for $\|BG^0(\omega)A^T\| < 1$ and, as a rule, with a sufficient accuracy we can restrict overselves to terms of order not higher than $(G^0)^4$. Then, taking into account the fact that

$$M\{G^0FG^0\} = F_dM\{(G^0)^2\}, \quad M\{G^0FG^0FG^0\} = F_d^2M\{(G^0)^3\}, \quad M\{G^0FG^0FG^0FG^0\} = F_d^3M\{(G^0)^4\} + \bar{U},$$

where F_d is a deterministic diagonal matrix formed from the elements f_{ii} of matrix F; and $\bar{U}$ is a deterministic mxm matrix with the diagonal elements $\bar{u}_{kk}$ $\sum_{i=1}^{m} f_{ki}^2f_{ii}M\{(g_k^0)^2\}\, M\{(g_i^0)^2\}$, $i \neq k$, and with the elements $\bar{u}_{kl} = (f_{kl}^3 + f_{kl}f_{kk}f_{ll})M\{(g_k^0)^2\}M\{(g_l^0)^2\}$, we obtain

$$\bar{W} = -F_d\bar{W}_1 - \bar{U}, \quad M\{W\bar{Q}W\} - \bar{W}_2\bar{Q}_d + \bar{W}_3,$$

where $\bar{W}_1$ is a diagonal deterministic matrix with the elements

$$\bar{w}_{1,ii} = M\{(g_{ii}^0)^2(1 + (f_{ii}g_{ii}^0)^3)/(1 + f_{ii}g_{ii}^0)\};$$

$\bar{W}_2$ is a diagonal deterministic matrix with the elements

$$\bar{w}_{2,ii} = M\{(g_{ii}^0)^2 - 2f_{ii}(g_{ii}^0)^3 + 3f_{ii}^2(g_{ii}^0)^4\};$$

$\bar{W}_3$ is a deterministic $m \times m$ matrix with the diagonal elements $\bar{W}_{3,kk}$ and $\bar{W}_{3,kl}$, which are equal to:

$$\bar{w}_{3,kk} = \sum_{i=1}^{m}(2f_{ki}f_{ii}\bar{q}_{ik} + f_{ki}^2\bar{q}_{ii})\, M\{(g_{kk}^0)^2\}\, M\{(g_{ii}^0)^2\}, \quad i \neq k,$$

$$\bar{w}_{3,kl} = (3f_{kl}^2\bar{q}_{kl} + f_{kk}f_{kl}\bar{q}_{ll} + f_{kk}f_{ll}\bar{q}_{kl} + f_{kl}f_{ll}\bar{q}_{kk})\, M\{(g_{kk}^0)^2\}\, M\{(g_{ll}^0)^2\};$$

and $\bar{Q}_D$ is a deterministic diagonal $m \times m$ matrix with the elements $\bar{q}_{ii}$ of matrix $\bar{Q}$.

Example of the Application of the Method. Consider a stochastic one-dimensional stationary temperature field of a body consisting of three contacting plates $l = l_1 + l_2 + l_3$ in length with different thermal conductivity coefficients and internal heat sources. The thermal conductivity coefficient for the midplate, $\lambda_2(x, \omega)$, is a random function with $M\{\lambda_2^0(x_i, \omega)\lambda_2^0(x_j, \omega)\} = 0$ for $i \neq j$ and each $x_i \in [0, l_2]$; $\lambda_2^0 = \lambda_2 - \bar{\lambda}_2$. For each $x_i \in [0, l_2]$, the thermal conductivities $\lambda_{2,i}(\omega)$ obey truncated normal distribution laws. The internal heat source power in the midplate, $f_2(x, (\omega)$, is a random function with the mathematical expectation f_2 and variance D_{f_2} and $M\{f_2^0x_i(\omega)f_2^0(x_i, \omega)\} = 0$ for $i \neq j$ and each $x_i \in [0, l_2]$, $f_2^0 = f - \bar{f}_2$. At the left boundary of the body, there occurs heat transfer with a medium which has temperature u_a and the random heat transfer coefficient $\alpha(\omega)$ obeying the uniform distribution law. The right boundary of the body is assigned the random temperature $u_e(\omega)$ with the mathematical expectation $\bar{u}_e$ and variance D_{u_e}. Such a problem arises when analyzing a composite rod, heat-insulated at the sides, and heated up by the electric current passing through it. If there is a statistical scatter in the second rod length, then this randomness may be simulated by random thermal conductivity $\lambda_2(x, \omega)$ with deterministic rod length.

The mathematical model takes on the form

$$\lambda_1\frac{\partial^2 u_1(x, \omega)}{\partial x^2} + f_1 = 0, \quad (x, \omega) \in (0, l_1) \times \Omega,$$

$$\frac{\partial}{\partial x}\left(\lambda_2(x, \omega)\frac{\partial u_2(x, \omega)}{\partial x}\right) + f_2(x, \omega) = 0, \quad (x, \omega) \in [0, l_2] \times \Omega,$$

$$\lambda_3 \frac{\partial^2 u_3(x,\ \omega)}{\partial x^2} + f_3 = 0, \quad (x,\ \omega) \in [0,\ l_3) \times \Omega,$$

$$\lambda_1 \frac{\partial u_1(x,\ \omega)}{\partial x} = \alpha(\omega)(u(x,\ \omega) - u_a), \quad x = 0, \quad \omega \in \Omega,$$

$$u_3(x,\ \omega) = u_e(\omega), \quad x = l, \quad \omega \in \Omega.$$

Dividing the body under consideration into nine intervals in such a way that each nodal point is contained in one control volume (as is shown in Figure 1) and integrating the mathematical-model equations over each interval, we obtain their discrete analog. The difference approximation of these equations may be written down in the matrix form (12)

$$H(\omega)\, u(\omega) = Af'(\omega) - AG(\omega)\, \varphi'(\omega),$$

where H(ω) is the stochastic matrix of the form:

$$H(\omega) = \begin{bmatrix} \alpha(\omega)+\delta_1 & -\delta_1 & & & & & & \\ -\delta_1 & \delta_1+\delta_2 & -\delta_2 & & & & & \\ & -\delta_2 & \delta_2+\delta_3 & -\delta_3 & & & & \\ & & -\delta_3 & \delta_3+\delta_4(\omega) & -\delta_4 & & & \\ & & & -\delta_4 & \delta_4(\omega)+\delta_5(\omega) & -\delta_5(\omega) & & \\ & & & & -\delta_5(\omega) & \delta_5(\omega)+\delta_6 & -\delta_6 & \\ & & & & & -\delta_6 & \delta_6+\delta_7 & -\delta_7 \\ & & & & & & -\delta_7 & \delta_7+\delta_8 \end{bmatrix};$$

f′(ω) and φ′(ω) are the stochastic vectors written as:

$$f'(\omega) = \begin{bmatrix} f_1 l_1 + f_2(\omega)\, l_2 + f_3(l_3 - \Delta_8) \\ f_1(l_1 - \Delta_0) + f_2(\omega)\, l_2 + f_3(l_3 - \Delta_8) \\ f_1(\Delta_2 + \Delta_3') + f_2(\omega)\, l_2 + f_3(l_3 - \Delta_8) \\ f_1\Delta_3' + f_2(\omega)\, l_2 + f_3(l_3 - \Delta_8) \\ f_2(\omega)(\Delta_4 + \Delta_5') + f_3(l_3 - \Delta_8) \\ f_2(\omega)\,\Delta_5' + f_3(l_3 - \Delta_8) \\ f_3(\Delta_6 + \Delta_7) \\ f_3\Delta_7 \\ 0 \end{bmatrix}, \quad \varphi'(\omega) = \begin{bmatrix} -u_a \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -u_e(\omega) \end{bmatrix};$$

A is the matrix of the form:

$$A = \begin{bmatrix} 1 & -1 & & & & & & & \\ & 1 & -1 & & & & & & \\ & & 1 & -1 & & & & & \\ & & & 1 & -1 & & & & \\ & & & & 1 & -1 & & & \\ & & & & & 1 & -1 & & \\ & & & & & & 1 & -1 & \\ & & & & & & & 1 & 1 \end{bmatrix};$$

G(ω) is a stochastic diagonal matrix with the diagonal elements equal to $\alpha(\omega)$, λ_1/h_1, λ_1/h_2, λ_1/h_3, $\lambda_2(\omega)/h_4$, $\lambda_2(\omega)/h_5$, λ_3/h_6, λ_3/h_7, and λ_3/h_8. Defining matrices entering into (13) and (14), we obtain vectors of the mathematical expectations $\bar{u}$ and variances D of stochastic temperatures at nodes 1, 2,..., 8 of the body. At node 9 the mathematical expectation of the stochastic temperature $\bar{u}_9 = \bar{u}_e$ and the variance of the stochastic temperature $I_9 = D_{u_e}$.

Figure 1 presents the calculations, performed by the suggested method, of dimensionless distributions of the mathematical expectation Θ, the r.m.s. deviation σ_Θ, and the upper boundary of the confidence interval $\Theta_c = \bar{u} + 1.96\,\sigma_e$ for the confidence probability 0.95 of the stochastic dimensionless temperature distribution $\Theta(x, u) = (u(x, \omega)-u_a)/u_a$. The dimensionless deterministic initial data have the following values: $\lambda_1/\bar{\lambda}_2 = 10$, $\lambda_3/\bar{\lambda}_2 = 37$, $l_1/l_2 = 3$, $l_3/l_2 = 4.5$ $f_1/\bar{f}_2 = f_3/\bar{f}_2 = 1$; and the probability characteristics of the random quantities $\alpha(\omega)$, $\lambda_2(x, \omega)$, $f_2(x, \omega)$, and $u_e(\omega)$ have the following dimensionless values: $d_\alpha/\bar{\alpha} = 0.8$, $d_{\lambda_2}/\bar{\lambda}_2 = 0.8$, $d_{f_2}/\bar{f}_2 = 1.0$, and $d_{u_e}/\bar{u}_c = 0.45$, where $d_\omega = \omega_{max} - \omega_{min}$ is the scatter of the random quantity ω.

Conclusion. The proposed numerical method allows one to determine the mathematical expectations and variances of a stochastic temperature field described by a three-dimensional stochastic heat conduction equation and by the first-, second-, and third-kind boundary conditions with random coefficients. The region, for which the probability characteristics of the stochastic temperature field are to be defined, may be arbitrary. On the basis of the present method it is possible to develop computer codes for analyzing stochastic temperature fields of complex objects.

NOTATION

$u(x, \omega)$, stochastic temperature field; Ω, space of elementary events ω; $R(\omega)$, stochastic matrix of the system; $G(\omega)$, stochastic diagonal matrix of the system parameters; A, matrix of incidences; $\bar{u}$, vector of mathematical expectations; K, C, correlation and covariance matrices.

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